Asymptotic fixed-confidence guarantees

Theorem 1. Let \( \varepsilon > 0 \) and \( \delta > 0 \). Combined with GLR, stopping (1), the EB-TC\(_s\) algorithm satisfies that, for all \( \varepsilon \in \mathbb{D}^k \) with mean \( \mu \),

\[
\mathbb{E}[T_{n,s}] \leq \inf_{\varepsilon \in \mathbb{D}^k} \left\{ T_{n,s}(\varepsilon) + 1, S_{n,s}(\varepsilon) \right\} + 2K^2,
\]

where

\[
\limsup_{n \to \infty} \frac{T_{n,s}(\varepsilon)}{\log(1/\delta)} \leq 2n(\varepsilon)T_{n,1/2}(\varepsilon) \text{ and } S_{n,s}(\varepsilon/2) = O(K^2)S_{n,1/2}(\varepsilon/2)^2 \log^2(\varepsilon/1).
\]

Key technical tool

Lemma 1. Let \( \varepsilon \in (0,1) \) and \( n > K \). Assume there exists a sequence of events \( A_n(\varepsilon, \delta) \subset \mathbb{D} \) and positive reals \( (D_n(\varepsilon, \delta))_{n \in \mathbb{N}} \) such that, for all \( t \in \{ K+1, \ldots, n \} \), under the event \( A_n(\varepsilon, \delta) \), there exists \( i \in \{ B_t, C_t \} \), such that \( T_n(i) < D_n(\varepsilon, \delta) \). Then,

\[
\sum_{t=K+1}^{n} T_n(i) \leq n \sum_{t=K+1}^{n} K / D_n(i).
\]

Beyond fixed-confidence guarantees

Anytime guarantees on the probability of \( \varepsilon \)-error and the expected simple regret.

Theorem 3. Let \( \varepsilon_n \to 0 \) and \( p(x) = x - \log x \). The EB-TC\(_s\) algorithm with fixed proportions \( \beta = 1/3 \) satisfies that, for all \( \varepsilon \in \mathbb{D}^k \) with mean \( \mu \), for all \( n > 5K^2/2 \),

\[
\forall \varepsilon \geq 0, \mathbb{P}(\varepsilon \in \mathbb{D}^k) \leq K^2(2+\log n)^2 \exp \left( -p \left( n - \frac{5K^2}{2}/\log(n) \right) \right),
\]

\[
\mathbb{E}[\mu - \mu_j] \leq K^2(2+\log n)^2 \sum_{i \in [\varepsilon]} (\Delta_{i} - \Delta_{i}(\varepsilon)) \exp \left( -p \left( n - \frac{5K^2}{2}/\log(n) \right) \right),
\]

where \( (H_{i}(\varepsilon, \mu))_{i \in [\varepsilon]} \) are such that \( H_{i}(\varepsilon, \mu) = K(2\Delta_{i} - 3\varepsilon_{i})^2 \) and \( K/\Delta_{i}^2 \leq H_{i}(\varepsilon, \mu) - K \min_{i \in [\varepsilon]} [2\Delta_{i}^2 + 3\varepsilon_{i}^2 + 3\varepsilon_{i}^2] \) for all \( i > 1 \).

Notation: distinct mean gaps \( 0 = \Delta_1 < \Delta_2 < \cdots < \Delta_{K+1} = +\infty \) where \( \Delta_{i} = (\mu_i - \mu_{i+1}) \).

Other guarantees: unverifiable sample complexity and cumulative regret.

Conclusion

1. Good empirical performance as regards the sample complexity and the simple regret. Easy to implement and computationally inexpensive algorithm.
2. Asymptotic and finite confidence upper bound on the expected sample complexity. Asymptotic \( \beta \)-optimality in \( \varepsilon \)-BAI for Gaussian distributions.
3. Anytime upper bounds on the uniform \( \varepsilon \)-error and the expected simple regret.